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THE HYDROFOIL CORPORATION

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ANNAPOLIS, MD.

TECHNICAL MEMORANDUM No. HM-22

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LONGITUDINAL STABILITY EQUATIONS FOR HYDROFOIL CRAFT WITH CONSTANT LIFT CONTROL SYSTEM

PREPARED FOR THE OFFICE OF NAVAL RESEARCH
WASHINGTON, D. C.

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Longitudinal Stability Equations for Hydrofoil Craft With Constant Lift Control System

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by

C. H. Kahr

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Summary

The longitudinal equations of motion have been developed for a tandem foil hydrofoil craft with a constant lift control on the forward hydrofoil. The roots of the characteristic stability equation have been interpreted in terms of the physical motions expected for the craft. It has been shown that several of the component stability modes involved are independent of one another and thus, simplified solutions of the general four degrees of freedom case can be achieved without loss of accuracy.

In particular, control inertia and mass unbalance have a negligible effect upon the hydrofoil craft motions. Also, the short period craft oscillation can be isolated in study by assuming constant forward velocity of the craft.

Numerical correlation has been based upon the "Lantern" configuration developed by The Hydrofoil Corporation.

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1. Cotter, William E., Jr.: Summary and Analysis of Data on Damping in Yaw and Pitch for a Number of Airplane Models. NACA T.N. 1080
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3. Kahr, Charles H.: General Longitudinal Stability Equations for Hydrofoil Systems with Specific Application To The Cape Cod Boat. Hydrofoil Corporation T.M.-21
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1) Introduction

It has been shown that hydrocraft can be considered to fall into two main classifications (ref. 3, 4, 5).

(1) those craft which have demonstrated inherent longitudinal stability due to hydrofoil configurations where submerged lifting area varies with depth; i.e. ladder hydrofoils, hydrofoils with dihedral (surface piercing), etc.

(2) those craft, employing completely submerged tandem hydrofoils, which seem to possess no apparent inherent stability. The use of either manual or automatic control is necessary to maintain a set submergence and attitude of the system.

The hydrofoils of category (1) give relatively low lift/drag ratios because of continual contact with the water- air interface. Thus, the craft of category (2) have a clear advantage if sustained flight can be achieved. This type of craft will be discussed in this report. A constant lift hydrofoil system is used to give automatic control of the craft.

2) Table of Symbols

Subscripts

A	aft hydrofoil
F	forward hydrofoil
o	initial or equilibrium value
c	craft (hydrocraft)
p	pilot hydrofoil of control system

Symbols

u, w	linear velocity components along x, z axes respectively
h	vertical displacement of hydrocraft center of gravity
θ	pitch angle
q	angular velocity $\quad q = \frac{d\theta}{dt}$
α	angle of attack
ϵ	downwash angle at hydrodynamic center of aft hydrofoil
V	resultant linear velocity of hydrocraft
X, Z	forces along x, z axes
M	moment in pitch about hydrocraft center of gravity
H.M.	hinge moment of forward control system about hinge line
h.c.	hydrodynamic center
C.G.	center of gravity
C_L, C_D	lift, drag coefficients of main hydrofoils
C_l, C_d	lift, drag coefficients of pilot hydrofoil

ρ density of water

W weight of hydrocraft

m mass of hydrocraft plus additional apparent mass of hydrofoils

m_c mass of hydrocraft

m_F mass of forward hydrofoil system (including control)

K_y radius of gyration in pitch for the hydrocraft

K_F radius of gyration of forward control system

ℓ distance between the hydrodynamic centers of fore and aft hydrofoils

ℓ_F, ℓ_A distance between hydrocraft c.g. and hydrofoil h.c.

S plan area

λ complex root of characteristic stability equation $= a + i\omega$

P period of oscillation $= \frac{2\pi}{\omega}$

$T_{1/2}$ time to damp to one-half the original amplitude $= \frac{\log 2}{a}$

$$\left.\begin{aligned} X_u &= \frac{1}{m_c}\frac{\partial X}{\partial u} \\ Z_q &= \frac{1}{m}\frac{\partial Z}{\partial q} \\ M_q &= \frac{1}{m}\frac{\partial M}{\partial q} \\ H_\theta &= \frac{1}{m_F}\frac{\partial H.M.}{\partial \theta} \end{aligned}\right\}$$ dimensional stability derivatives

3) Description of Hydrofoil System with Constant Lift Control

Zero dihedral hydrofoils operating at submergences of approximately 2-1/2 chords possess negligible depth stability. One of the possibilities for rectifying this deficiency involves the use of the so-called "constant lift hydrofoil" in conjunction with a pilot foil. In this system, a tendency towards maintenance of a set submergence is accomplished through the use of a surface sensing or pilot foil which controls the main hydrofoil's angle of attack. It is considered necessary to employ this arrangement upon only one hydrofoil (forward) of the tandem combination.

Referring to the figure on p. 16, it is seen that a simple lever, composed of pilot foil, connecting rod, and the main foil, is used. An increase in depth, for example, results in increased lift and drag forces for the pilot foil and a corresponding clockwise moment imposed about the hinge line of the system. This tends to increase the angle of attack of the main foil and increase its lift to return the system to the equilibrium depth.

It is noted that static balance is obtained by means of the force "F". This force can be changed to provide various equilibrium depths.

Since the moment about the hydrodynamic center is a function of velocity for unsymmetrical profiles, it is necessary to specify a symmetrical section.

4) Longitudinal Equations of Motion

The longitudinal equations of motion are developed from the freedom of the craft to pitch, heave, and change horizontal speed. Heretofore, it has been customary to assume constant forward speed when analyzing craft motions (ref. 3,4,5). This assumption needs investigation and thus the analysis here considers three degrees of freedom for the boat. Due to the type of automatic control of submergence used here (constant lift system), the control degree of freedom must also be included.

From ref. 1, the equations of motion for a hydrocraft are:

$$
\begin{aligned}
\dot{u} + w_0 q + (g\cos\theta_0)\theta - X_u u - X_w w - X_q q - X_h h - X_\theta \theta - X_{\alpha_F}\alpha_F &= 0 \\
\dot{w} - u_0 q + (g\sin\theta_0)\theta - Z_u u - Z_w w - Z_q q - Z_h h - Z_\theta \theta - Z_{\alpha_F}\alpha_F &= 0 \qquad (1) \\
k_y^2 \dot{q} - M_u u - M_w w - M_q q - M_h h - M_\theta \theta - M_{\alpha_F}\alpha_F &= 0
\end{aligned}
$$

The equation corresponding to the control degree of freedom is:

$$
\frac{\partial H.M.}{\partial q} q + \frac{\partial H.M.}{\partial \theta}\theta + \frac{\partial H.M.}{\partial h} h + \frac{\partial H.M.}{\partial \alpha_F}\alpha_F + \frac{\partial H.M.}{\partial \ddot{\alpha}_F}\ddot{\alpha}_F + \frac{\partial H.M.}{\partial u} u = \left(I_{FOIL} + I_{CONTROL}\right)\left(\ddot{\alpha}_F + \ddot{\theta}\right) - m_F x_F l_F \ddot{\theta} \qquad (2)
$$

where H.M.= moment about hinge line of forward foil system

m_f = mass of forward foil and control system (including additional apparent mass)

x_f = distance from c.g. of forward system to hinge line

l_f = distance from c.g. of craft to c.g. of forward system (assumed same as distance from c.g. of craft to hydrodynamic center of forward foil)

Equation (2) can be put in the form of eq's (1) :

$$K_f^2(\ddot{\alpha}_f+\ddot{\theta})-x_f l_f\ddot{\theta}-H_u u-H_q q-H_h h-H_\theta\theta-H_{\alpha_f}\alpha_f-H_{\dot{\alpha}_f}\dot{\alpha}_f=0 \quad (3)$$

note that in eq's (1) :

$$Z_w=\frac{1}{m}\frac{\partial Z}{\partial w} \quad ; \quad M_q=\frac{1}{m}\frac{\partial M}{\partial q} \quad \text{ETC.}$$

$$m = m_{CRAFT} + m_{APPARENT}$$

and in equation (3) :

$$H_w=\frac{1}{m_f}\frac{\partial H.M.}{\partial w} \quad \text{ETC.}$$

It is assumed that the main hydrofoils will operate at depths of approximately 2.5 chords where hydrodynamic forces are not changed appreciably with changes in depth. This means that the stability derivatives Z_h, Z_θ, M_h, M_θ are assumed negligible.

However, the surface effects on the pilot foil of the constant lift system will be considered; i.e. H_h, H_θ are included.

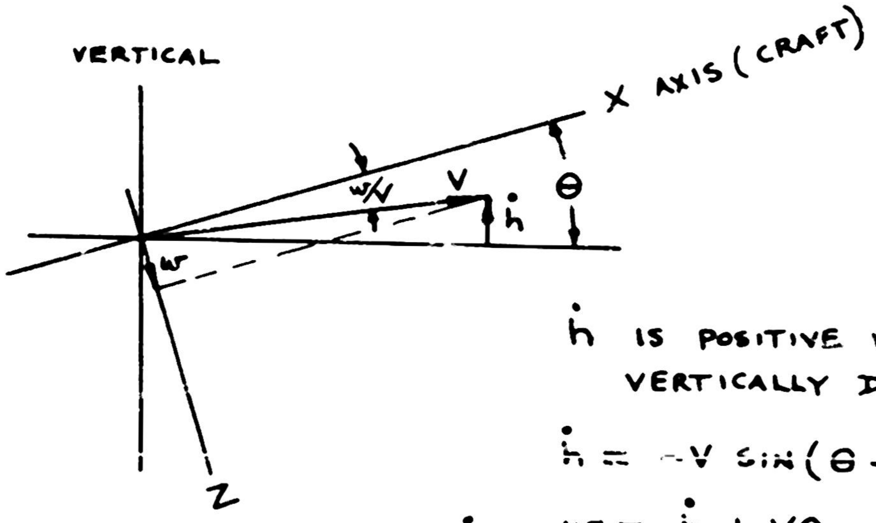
The initial trim angle of craft, θ_o , is set equal to zero.

In the hinge moment equation, the term $x_f l_f\ddot{\theta}$ corresponds to mass unbalance of the control system about the hinge line.

Restated and simplified in accordance with the assumptions, the longitudinal equations are:

$$
\begin{aligned}
&\dot{u} - X_u u - X_w w - X_q q - X_{\alpha_F}\alpha_F + g\theta = 0 \\
&\dot{w} - u_0 q - Z_u u - Z_w w - Z_q q - Z_{\alpha_F}\alpha_F = 0 \qquad (4) \\
&k_y^2 \dot{q} - M_u u - M_w w - M_q q - M_{\alpha_F}\alpha_F = 0 \\
&k_F^2(\ddot{\alpha}_F + \ddot{\theta}) - x_F l_F \ddot{\theta} - H_u u - H_q q - H_h h - H_\theta \theta - H_{\alpha_F}\alpha_F - H_{\dot{\alpha}_F}\dot{\alpha}_F = 0
\end{aligned}
$$

The main variables are considered to be u, h, θ, α_F. It is necessary to use h (vertical displacement) instead of z because any type of automatic control will attempt to sense absolute depth rather than a distance along the Z axis of the craft which at an arbitrary instant may be pitched through an angle θ.



$\dot{h}$ IS POSITIVE WHEN DIRECTED VERTICALLY DOWNWARD

$$\dot{h} = -V \sin\left(\theta - \frac{w}{V}\right) \cong w - V\theta$$

$$\therefore \quad w = \dot{h} + V\theta \qquad (5)$$

Substituting $u = C_1 e^{\lambda t}$; $h = C_2 e^{\lambda t}$; $\theta = C_3 e^{\lambda t}$; $\alpha_F = C_4 e^{\lambda t}$ in the equations (4), (5) and solving by determinants will yield the characteristic stability equation. The hydrocraft's motions following a disturbance can be determined from the roots of this auxiliary equation.

Therefore:

$$
\begin{aligned}
(\lambda - X_u)C_1 &- X_w \lambda C_2 + (g - X_w V - X_q \lambda)C_3 - X_{\alpha_F} C_4 = 0 \\
-Z_u C_1 &+ (\lambda^2 - Z_w \lambda)C_2 + (V\lambda - Z_w V)C_3 - Z_{\alpha_F} C_4 = 0 \\
-M_u C_1 &- M_w \lambda C_2 + (K_y^2 \lambda^2 - M_q \lambda - M_w V)C_3 - M_{\alpha_F} C_4 = 0 \\
-H_u C_1 &- H_h C_2 + (K_F^2 \lambda^2 - x_F \ell_F \lambda^2 - H_q \lambda - H_\theta)C_3 + (K_F^2 \lambda^2 - H_{\dot{\alpha}_F} \lambda - H_{\alpha_F})C_4 = 0
\end{aligned}
$$

The characteristic stability equation for this general case of four degrees of freedom is the septic equation:

$$A\lambda^7 + B\lambda^6 + C\lambda^5 + D\lambda^4 + E\lambda^3 + F\lambda^2 + G\lambda + H = 0$$

where the coefficients A, B, C, etc. are functions of the hydrodynamic and mass parameters of the hydrocraft.

Appendix A gives these characteristics for the "Lanterns" hydrocraft and Appendix B gives the stability derivatives required for this configuration.

Interpretation of the Roots of the Auxiliary Equation

The longitudinal equations of motion for the general case of four degrees of freedom result in a characteristic stability equation of the seventh degree. The problem here is to interpret the roots of this equation in terms of the physical motion of the hydrocraft following a disturbance, and to see if one or more of the component motions can be disregarded in the consideration of the remainder. This is necessary in order to determine possible legitimate approximations for use in reducing the complexity of the equations of motion.

Using the numerical data given in Appendices A and B, the following cases were analyzed.

Four Degrees of Freedom with Control Inertia and Mass Unbalance Included

The roots of the septic auxiliary equation occur as either three real roots and two pairs of complex roots or one real root and three pairs of complex roots. The grouping of the roots is dependent upon the craft's c.g. location and dimensions of the hydrofoils.

Typical values for the roots are:

$$\lambda_{1,2} = -31.2 \pm 17.93\,i$$

$$\lambda_{3,4} = -.61 \pm 7.30\,i$$

$$\lambda_{5,6} = +.3 \pm .51\,i \quad \left\{ \text{CONFIGURATION CHANGES MAY RESULT IN RESOLUTION INTO A PAIR OF REAL ROOTS} \right.$$

$$\lambda_{7} = -19.1$$

The damping and period of the craft motions following a disturbance is given by the equations:

$$T_{1/2 \text{ amplitude}} \text{ (sec.)} = \frac{\log 2}{-a} \quad ; \quad P \text{ (sec.)} = \frac{2\pi}{\omega}$$

where $\lambda = a \pm i\omega$ For the present case, the stability modes have the characteristics:

$$\lambda_{1,2} \qquad T_{1/2} = .022 \text{ SEC} \quad ; \quad P = .35 \text{ SEC.} \quad \text{(CONTROL)}$$

$$\left.\begin{array}{llll} \lambda_{3,4} & T_{1/2} = 1.14 \text{ SEC.} & ; & P = .86 \text{ SEC.} \\ \lambda_{5,6} & T_{2} = 2.31 \text{ SEC.} & ; & P = 12.3 \text{ SEC.} \\ \lambda_{7} & T_{1/2} = .036 \text{ SEC} & ; & P = \infty \end{array}\right\} \text{(CRAFT)}$$

It has been determined that the very highly damped, short period oscillation corresponds to the constant lift control system. After a disturbance in pitch or heave, the control oscillation damps out and then the control floats at the angle of attack consistent with the attitude of the craft during the continuing craft oscillations. It is noted that there are two craft oscillations involved here. It appears as though the diverging, long period motion may be altered considerably in testing the craft due to the large amplitude displacements occurring. The equations as set down here are valid only for small displacements .

As is shown in the next section, this diverging oscillation disappears when the craft velocity is assumed constant.

Three Degrees of Freedom With and Without Control Mass Effects

The characteristic stability equation can be reduced to a quintic by assuming the craft velocity to be a constant. The degree of the equation can be further reduced by assuming the control inertia and mass unbalance to equal zero. A comparison of the roots for these cases against the previous four degrees of freedom case shows that the component modes are virtually independent of one another.

four degrees of freedom control inertia and mass unbalance		three degrees of freedom control inertia and mass unbalance	
included		included	deleted
$\lambda = -31.2 \pm 17.93\,i$	(CONTROL)	$\lambda = -31.5 \pm 18.\,i$	$\lambda = -40.3$
$\lambda = -.61 \pm 7.30\,i$	(CRAFT)	$\lambda = -.74 \pm 8.2\,i$	$\lambda = -1.32 \pm 8.4\,i$
$\lambda = +.3 \pm .51\,i$	(CRAFT)	$\lambda =$ ——	$\lambda =$ ——
$\lambda = -19.1$	(CRAFT)	$\lambda = -19.0$	$\lambda = -18.4$

It is seen that the relatively unimportant control oscillation degenerates to a simple subsidence mode when control inertia and mass unbalance are assumed equal to zero.

When correlating the actual test craft motions with those predicted by this theory, it will be possible to use the above simplifications to expedite the study.

APPENDIX A

Data for "Lantern" Configuration

The basic "Lantern" configuration consists of a tandem foil hydrofoil craft with a constant lift system on the forward hydrofoil. Although the stability equations have been calculated for a c.g. position exactly midway between the equal area fore and aft hydrofoils, positive static stability exists (since the stability derivative M_w is negative).

Weight = 6000 lbs. $\qquad$ $V = 20$ Ft./sec.

Area Fwd. Foil $\quad S_F = 20$ Sq. Ft. $\qquad$ Chord = 1.0 Ft.

Area Aft Foil $\quad S_A = 20$ Sq. Ft. $\qquad$ Fore & Aft

Distance Between Foils $\quad l = 25$ ft. $\qquad$ $C_{L_F} = C_{L_A} = .375$

C.G. to Fwd. Foil $\quad l_f = 12.5$ ft.

C.G. to Aft Foil $\quad l_A = 12.5$ ft. $\qquad$ $\left(\frac{dC_L}{d\alpha}\right)_A = \left(\frac{dC_L}{d\alpha}\right)_F = 4.75$/RAD.

$(\text{Craft Radius of Gyration})^2 = K_Y^2 = 90 \text{ Ft.}^2$

$(\text{Control Radius of Gyration})^2 = K_F^2 = .25 \text{ Ft.}^2$

$M_C = \text{Mass of Craft} = \frac{6000}{g} = 187 \text{ Slugs}$

$M = M_C + 2\rho \frac{\pi}{4} Sc \text{ (Additional Apparent Foil Mass)} = 249 \text{ Slugs}$

$M_F = \frac{200}{g} + \rho \frac{\pi}{4} Sc = 38 \text{ Slugs}$

Fwd. Control System Dimensions (See Fig. Appendix B)

$a = .667$ ft. ; $\quad b = .166$ ft. ; $\quad e = 3.0$ ft. ; $S_p = 1.0$ Sq.Ft.

$\frac{d\varepsilon}{d\alpha} = -.10/\text{Radian}$ $\quad$ (Formula Ref. 3 P. 24)

The values for the parameters $\left(\frac{dC_l}{d\alpha}\right), \left(\frac{dC_d}{d\alpha}\right), \left(\frac{dC_l}{dh}\right), \left(\frac{dC_d}{dh}\right)$ are those consistent with a surface piercing pilot hydrofoil close to the water surface (one-quarter to one-half chord). These values, therefore, represent extreme effectiveness as far as surface sensing is concerned.

$$\left(\frac{dC_l}{d\alpha}\right)_p = .12/\text{RADIAN} \qquad \left(\frac{dC_l}{dh}\right)_p = .450/\text{FT.}$$

$$\left(\frac{dC_d}{d\alpha}\right)_p = .33/\text{RADIAN} \qquad \left(\frac{dC_d}{dh}\right)_p = 2.0/\text{FT.}$$

This data was estimated from Stevens E.T.T. reports and NACA T.R. 918.

APPENDIX B

Longitudinal Stability Derivatives

The Z and M derivatives can be set down without alteration from ref. 3. This procedure is consistent with the assumption that force changes on the pilot foil result in a negligible contribution to the main craft derivatives. These pilot foil forces are considered, of course, when evaluating the control hinge moment derivatives.

$$Z_{\alpha_F} = -\frac{1}{2}\frac{\rho V^2 S_F}{m}\left(\frac{dC_L}{d\alpha}\right)_F$$

$$Z_w = -\frac{1}{2}\frac{\rho V S_F}{m}\left[\left(\frac{dC_L}{d\alpha}\right)_F + \frac{S_A}{S_F}\left(1-\frac{d\epsilon}{d\alpha}\right)\left(\frac{dC_L}{d\alpha}\right)_A\right]$$

$$Z_q = \frac{1}{2}\frac{\rho V S_F}{m}\left[\left(\frac{dC_L}{d\alpha}\right)_F \ell_F - \frac{S_A}{S_F}\ell_A\left(1-\frac{\ell}{\ell_A}\frac{d\epsilon}{d\alpha}\right)\left(\frac{dC_L}{d\alpha}\right)_A\right]$$

$$M_{\alpha_F} = \frac{1}{2}\frac{\rho V^2 S_F}{m}\left[\left(\frac{dC_L}{d\alpha}\right)_F \ell_F - d_F\left(\frac{dC_D}{d\alpha}\right)_F\right]$$

$$M_w = \frac{1}{2}\frac{\rho V S_F}{m}\left\{\ell_F\left(\frac{dC_L}{d\alpha}\right)_F + d_F\left[C_{L_F} - \left(\frac{dC_D}{d\alpha}\right)_F\right]\right.$$
$$\left. + \left(1-\frac{d\epsilon}{d\alpha}\right)\frac{S_A}{S_F}\left[d_A\left(C_{L_A} - \left(\frac{dC_D}{d\alpha}\right)_A\right) - \left(\frac{dC_L}{d\alpha}\right)_A \ell_A\right]\right\}$$

$$M_q = \frac{1}{2}\frac{\rho V S_F}{m}\left[\left(\frac{dC_D}{d\alpha}\right)_F \ell_F d_F - \left(\frac{dC_D}{d\alpha}\right)_A \ell_A d_A \frac{S_A}{S_F}\left(1-\frac{\ell}{\ell_A}\frac{d\epsilon}{d\alpha}\right)\right.$$
$$\left. - \left(\frac{dC_L}{d\alpha}\right)_F \ell_F^2 - \frac{S_A}{S_F}\ell_A^2\left(\frac{dC_L}{d\alpha}\right)_A\left(1-\frac{\ell}{\ell_A}\frac{d\epsilon}{d\alpha}\right)\right]$$

The craft derivatives not considered heretofore (ref. 3) are: Z_u, M_u, X_{α_F}, X_u, X_w, X_q. By methods of ref. 3:

$$Z_u = -(C_{L_F} S_F + C_{L_A} S_A)\frac{\rho V}{m}$$

$$M_u = \frac{\rho V}{m}\left[C_{L_F} S_F \ell_F - C_{L_A} S_A \ell_A - C_{D_F} S_F d_F - C_{D_A} S_A d_A\right]$$

$$X_{\alpha_F} = -\frac{1}{2}\frac{\rho V^2 S_F}{m_c}\left(\frac{dC_D}{d\alpha}\right)_F$$

$$X_u = -\frac{\rho V}{m_c}\left(C_{D_F} S_F + C_{D_A} S_A\right)$$

$$X_w = -\frac{1}{2}\frac{\rho V S_F}{m_c}\left[\left(\frac{dC_D}{d\alpha}\right)_F + \frac{S_A}{S_F}\left(1-\frac{d\epsilon}{d\alpha}\right)\left(\frac{dC_D}{d\alpha}\right)_A\right]$$

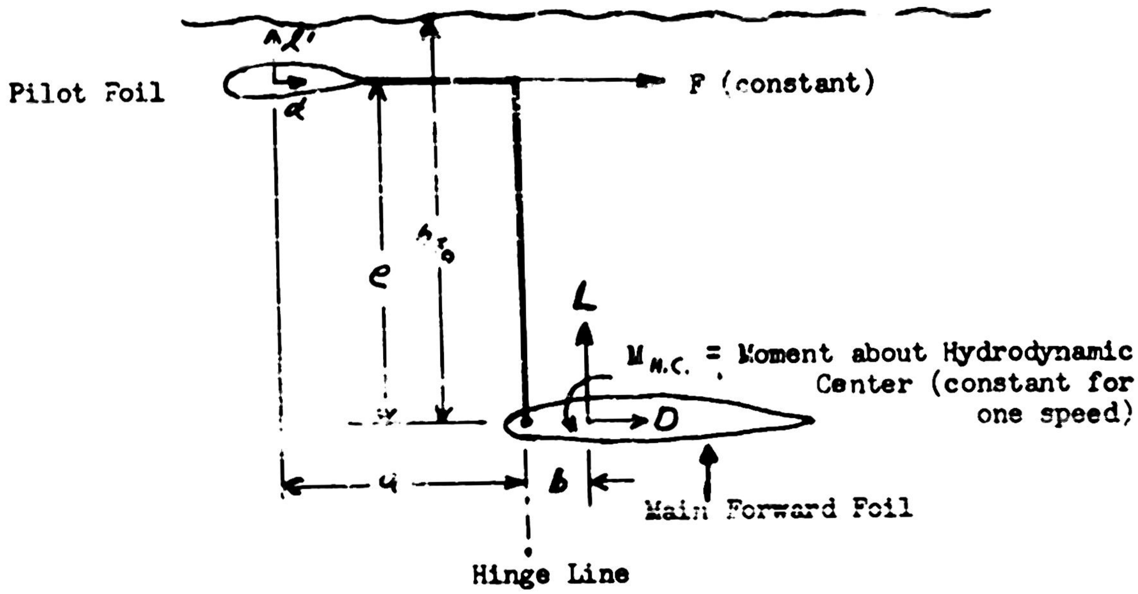
$$X_q = \frac{1}{2}\frac{\rho V S_F}{m_c}\left[\left(\frac{dC_D}{d\alpha}\right)_F \ell_F - \frac{S_A}{S_F}\left(1-\frac{\ell}{\ell_A}\frac{d\epsilon}{d\alpha}\right)\left(\frac{dC_D}{d\alpha}\right)_A\right]$$

$$m = m_{CRAFT} + m_{APPARENT}$$

$$m_c = m_{CRAFT}$$

It should be noted that the Z and M derivatives are using additional apparent mass terms consistent with foil accelerations which are approximately perpendicular to the chord line. For the X derivatives the additional apparent mass is assumed equal to zero.

The basic setup for the constant lift system is shown in the sketch given below. It is desired to know how the hinge moments change with changes in the variables α_F, u, h, θ (and their derivatives.



Pilot foil, connecting rod, and main foil are a rigid system pivoting about hinge line.

For $h_{F_0} > 2.5$ chords, a submergence change results in lift and drag changes on pilot foil only.

H_{α_F}

$$\Delta\, H.M. = \left(\frac{dC_l}{d\alpha}\right)_p \Delta\alpha_F \tfrac{1}{2}\rho V^2 S_p a + \left(\frac{dC_d}{d\alpha}\right)_p \Delta\alpha_F \tfrac{1}{2}\rho V^2 S_p e - \left(\frac{dC_L}{d\alpha}\right)_F \Delta\alpha_F \tfrac{1}{2}\rho V^2 S_F b$$

$$H_{\alpha_F} = \frac{1}{m_F}\frac{\partial H.M.}{\partial \alpha_F} = \frac{1}{2}\frac{\rho V^2}{m_F}\left[\left(\frac{dC_l}{d\alpha}\right)_p S_p a + \left(\frac{dC_d}{d\alpha}\right)_p S_p e - \left(\frac{dC_l}{d\alpha}\right)_F S_F b\right]$$

$H_{\dot{\alpha}_F}$ The damping in pitch of an airfoil can be derived from data given T.N. 1080 (ref. 1), and Durand (ref. 2). The tests in T.N. 1080 give damping derivatives with pivot at the one-quarter chord point. Durand's data can be used for any pivot from near the leading edge to approximately the one-half chord point of the foil. For the same pivot point (one-quarter chord point), the two references are in agreement.

For the case used here (pivot at the one-twelfth chord point):

$$H_{\dot{\alpha}_F} = \frac{1}{m_F}\frac{\partial H.M.}{\partial \dot{\alpha}_F} = -\frac{1}{m_F}(.45)\rho V S_F C_F^2$$

H_u

$$H_u = \frac{1}{m}\frac{\partial H.M.}{\partial u} = \frac{\rho V}{m_F}\Big[-C_{L_F} S_F b + C_{l'} S_p a + C_d S_p e - C_{m_{A.C.}} S_F C_F\Big]$$

H_h An increase in submergence (+h) results in a positive hinge moment upon the system.

$$\Delta H.M. = \Delta C_{l'} \tfrac{1}{2}\rho V^2 S_p a + \Delta C_d \tfrac{1}{2}\rho V^2 S_p e$$

$$= \frac{dC_{l'}}{dh} h \tfrac{1}{2}\rho V^2 S_p a + \frac{dC_d}{dh} h \tfrac{1}{2}\rho V^2 S_p e$$

$$H_h = \frac{1}{m_F}\frac{\partial H.M.}{\partial h} = \frac{1}{2}\frac{\rho V^2 S_p}{m_F}\Big[\frac{dC_{l'}}{dh} a + \frac{dC_d}{dh} e\Big]$$

Numerical Valu For The Stability Derivatives

$X_u = -.\ 35$	$Z_u = -2.41$
$X_w = -\ 0$	$Z_w = -16.00$
$X_q = -\ 17$	$Z_q = -19.08$
$X_{\alpha_F} = -\ .56$	$Z_{\alpha_F} = -152.70$

$M_u = \ 39$	$H_u = -10.21$
$M_w = -\ .00$	$H_h = +66.40$
$M_q = -\ 20.0$	$H_\theta = -864.0$
$M_{\alpha_F} = +\ 370.0$	$H_q = +96.5$
	$H_{\alpha_F} = -154.2$
	$H_{\dot{\alpha}_F} = -9.5$